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Quasi-Periodic Gaits of Passive One-Legged Hopper

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Abstract

In this paper, we present a novel controller for a passive one-legged hopping robot. First, based on the dynamics of this nonlinear hybrid system, we derive a simple control law to ensure the total energy preservation and continuation. Simulation results show that the robot can hop from the wide set of initial conditions. The generated hopping gaits are found to be quasi-periodic orbits, which can be seen in some Hamiltonian systems. Next, we propose a simple parameter adaptation law to asymptotically stabilize the quasi-periodic gaits to the periodic gaits of arbitrary period, and spring stiffness adaptation law to minimize control inputs. Simulation results show that the robot eventually hops without any control inputs, especially for 1-periodic gait. We believe that the controllers we invented have much potential for energy-efficient control of legged running robot.

1 Introduction

After the Raibert's excellent works [1], one-legged hopping robots attached with only the leg spring, have been widely studied both experimentally [2, 3, 4, 5] and theoretically [6, 7, 8].

In addition to the leg spring, hip spring also plays important role for animal running [9]. It enables the leg to be swung passively. Tompson and Raibert showed that spring-driven one-legged hopping robot shown in **Figure 1**, can hop without any inputs, provided if the initial conditions are appropriately chosen [10]. Therefore, this model is a good template model for the purpose of studying energy-efficient running. Since this model is shown to be marginally stable and eventually falls without controls, some suitable controller should be applied to ensure the stability.

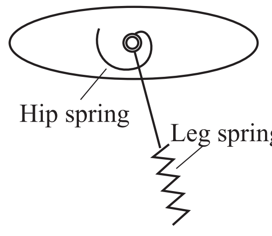

Figure 1: *Passive one-legged hopper*

Ahmadi and Buehler applied Raibert's celebrated *Foot Placement Algorithm* [1] to this passive hopping robot, in which the *Neutral Point* should be pre-approximated. They realized energy-efficient hopping in simulation and experiment [11, 12]. On the other hand, François and Samson derived new controller different from Raibert's [13]. They applied general control method used in nonlinear oscillatory system. That is, first constructing Poincare map (discrete system), then linearizing it around fixed points, and finally applying some linear feedback to get asymptotically stable periodic orbit. However, since the model cannot be integrable, both examples above need approximated models, or approximated periodic solutions, to derive the controllers. Therefore, there remains static error comes from the modeling error.

In this paper, inspired from François and Samson's work [13], we present a novel controller to realize energy-efficient one-legged hopping. Instead of depending on some pre-planned periodic solutions, or target dynamics, here we utilize the intrinsic dynamics of the original nonlinear hybrid system to make the robot generate natural hopping gaits. A model description is the same as those in [13] and reviewed in Section 2. Since our goal is energy-efficient hopping, we explore the condition of energy-preservation and continuation in Section 3. Then, we derive a control law to ensure those conditions and show the simulation resuts in Section 4. In Section 5, two parameter adaptation laws, one for asymptotic stabilization to desired periodic gait, and the other for input minimization, are given.

Simulation results shows that the controllers we invented have much potential for energy-efficient control of legged running robot.

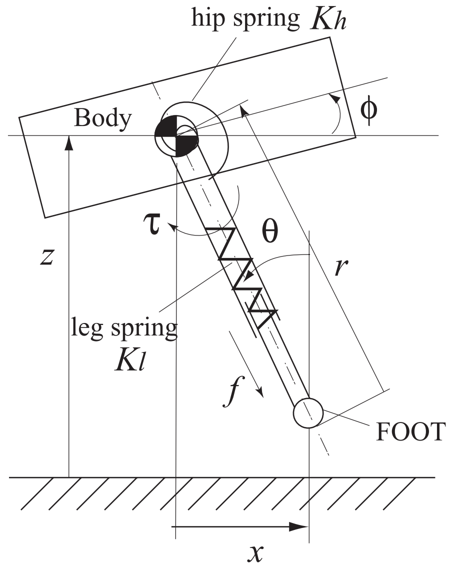

Figure 2: *Model definition*

2 Model description of passive hopping robot

We consider exactly the same model of passive hopping robot as [13] in this paper. In this section, the model description is reviewed.

2.1 Model definition and notation

We consider the planar one-legged hopping robot shown in **Figure 2**. The robot is attached with not only the leg spring but also hip spring.

We impose the following assumptions on the model as seen in many related literatures.

(A) The center of mass (COM) of the body is just on the hip joint and COM of the leg lies on the leg

(B) Mass of the foot (unsprung mass) is negligible

(C) The springs are mass-less and non-dissipating

(D) The foot does not bounce back, nor slip the ground (inelastic impulsive impact)

In addition to these assumptions, we further suppose, just for the simplicity, that the COM of the robot is on the hip joint without loss of generality. **Table 1** summarizes the variables appear in this paper. The equations of motion are composed of four phases; stance phase, lift-off phase, flight phase, and touch-down phase. **Table 2** defines the phase-indicating suffix of the variables. For example, $\dot{x}_{lo}$ represents the forward velocity of COM at the lift off. **Table 3** shows the physical parameters, together with the values used in later simulations.

Table 1: *Variables*

	Meaning	Unit
x	horizontal position of hip	m
z	vertical position of hip	m
r	leg length	m
ϕ	body angle	rad
θ	leg angle	rad
f	applied force of leg	N
τ	applied torque of hip	N
μ	energy dissipation	
E	total mechanical energy	J
T_s	stance time	s
T_v	flight time	s

Table 2: *Phase-indicating suffix*

Suffix	Meaning
td-	just before touch-down
td+	just after touch-down
lo	lift-off

Table 3: *Physical parameters*

	Meaning	Unit	Value
g	gravity acceleration	m/s^2	9.8
M	total mass	kg	12
r_0	natural leg length	m	0.5
J_b	body inertia about hip	kgm^2	0.5
J_l	leg inertia about hip	kgm^2	0.11
K_l	leg spring stiffness	N/m	3000
K_h	hip spring stiffness	Nm/rad	10

2.2 Equation of motion

At the stance phase, the leg compresses and extends, and the angular momentum of the robot around the contact point evolutes under the gravity field.

$$\left\{\begin{array}{l} M\ddot{r} + K_l(r - r_0) - Mr\dot{\theta}^2 = Mg(1 - \cos\theta) + f \\ J_l\ddot{\theta} + J_b\ddot{\phi} + \frac{d}{dt}(Mr^2\dot{\theta}) = rMg\sin\theta \\ J_b\ddot{\phi} + K_h(\theta - \phi) = \tau_s \end{array}\right. \quad (1)$$

Here, f is the control force to the leg, while τ_s is the control torque to the hip joint, which is applied during stance. The spring is initially loaded with the same value of gravity force (Mg).

At the flight phase, COM of the robot moves along the ballistic flight path and the angular momentum of the robot around the COM is preserved.

$$\begin{cases} \ddot{x} = 0 \\ \ddot{z} = -g \\ J_l\ddot{\theta} + J_b\ddot{\phi} = 0 \\ J_b\ddot{\phi} + K_h(\theta - \phi) = \tau_v. \end{cases} \tag{2}$$

Here, τ_v represents the control torque to the hip joint, which is applied during flight.

By the assumption (D) in Section 2.1, the velocities of the generalized coordinates change instantaneously at the touch-down phase, according to the following equations.

$$\begin{cases} \dot{x}_{td+} = \dot{x}_{td-} - \frac{J_l \cos\theta_{td}}{J_l + Mr_0^2}\mu_{td-} \\ \dot{z}_{td+} = \dot{z}_{td-} - \frac{J_l \sin\theta_{td}}{J_l + Mr_0^2}\mu_{td-} \\ \dot{\theta}_{td+} = \dot{\theta}_{td-} - \frac{Mr_0}{J_l + Mr_0^2}\mu_{td-} \\ \dot{\phi}_{td+} = \dot{\phi}_{td-} \\ \dot{r}_{td+} = \dot{z}_{td+}\cos\theta_{td} - \dot{x}_{td+}\sin\theta_{td}. \end{cases} \tag{3}$$

Here,

$$\mu_{td-} := \dot{x}_{td-}\cos\theta_{td} + \dot{z}_{td-}\sin\theta_{td} + r_0\dot{\theta}_{td-}. \tag{4}$$

At the lift-off phase, there is no discontinuous changes except for $\dot{r}_{lo} = 0$.

3 Analysis on the energy preservation and continuation

For a passive running robot, the analysis on the energy is important because, during complete passive running, the total mechanical energy is conserved.

According to (3), the energy change between just before touch-down and just after touch-down is calculated to be

$$\begin{aligned} & E_{td+} - E_{td-} \\ & = \left[\frac{1}{2}M(\dot{x}^2 + \dot{z}^2) + \frac{1}{2}J_b\dot{\phi}^2 + \frac{1}{2}J_l\dot{\theta}^2\right]_{td-}^{td+} \\ & = -\frac{MJ_l}{2(J_l + Mr_0^2)}\mu_{td-}^2. \end{aligned} \tag{5}$$

We call μ_{td-} "*Energy Dissipation Coefficient*" because, if the condition

$$\mu_{td-} = 0 \tag{6}$$

holds at touch-down and no input is applied to the robot, then total mechanical energy of the system is preserved during hopping. Of course if we apply some control inputs, then the internal energy is not conserved. But we can say that (6) is necessary condition for complete passive running.

Next, we consider the conditions for the robot to sustain hopping without falling to the ground. We can see the condition is found to be

$$\dot{r}_{td+} < 0. \tag{7}$$

This is the condition for the axial velocity of the leg just after touch-down to be negative. This is well understood if we recognize from Section 2.2 that the necessary condition for the robot to be in the stance phase is $r < r_0$. Unless this condition holds, the hybrid system cannot switch to the stance phase from the flight phase, and hence, it cannot continue time evolution. Note that (7) is not always satisfied.

4 "Non-Dissipative Touch-Down Control" and quasi-periodic hopping gait

4.1 Control goal

Due to the compactness of the phase space, if the continuous time evolution is ensured and the total energy is preserved, the solution of this hybrid nonlinear system lies on periodic orbits on the energy-invariant manifold. This invariant *hybrid flow* is the gait we are searching for. Therefore, we derive controller to ensure the both condition (6) and (7), and call this new controller "*Non-dissipative Touch-down Control*".

We can do this by applying control inputs only at the flight phase, and allowing the robot freely to move at stance phase with zero inputs. The reason why we are doing so is that if no energy dissipation occurs at touch-down ($\mu_{td-} = 0$), there is no interaction between the robot and environment, even we use apply control inputs at flight phase. Then, the total mechanical energy of the robot "including power source of the actuators", is exactly preserved. Energy preserving gait in this sense, is what we want.

Since we are using the control torque only at the flight phase, we define the control input τ as follows.

$$f = \tau_s = 0, \tag{8}$$

$$\tau_v =: \tau. \tag{9}$$

Our control problem is to find the control input τ that makes the robot land at the time T_v, with $(\theta_{td}, \dot{\theta}_{td-})$ satisfying (6) and (7), for any given lift-off states $(\dot{x}_{lo}, \theta_{lo}, \dot{\theta}_{lo}, \phi_{lo}, \dot{\phi}_{lo})$. Though this is the dead-beat control to bring T_v, θ_{td}, and $\dot{\theta}_{td-}$ to the desired values, we have only to choose some T_v and θ_{td}, because $\dot{\theta}_{td-}$ is automatically calculated by (6). There are, however, a large number of such pairs (T_v, θ_{td}) satisfying (7). Here, we choose the simplest values as follows.

First, we determine T_v so that the vertical velocity at touch-down and lift off is the same in magnitude and opposite in direction,

$$T_v = \frac{2\dot{z}_{lo}}{g}. \tag{10}$$

Next, we choose desired touch-down angle $\bar{\theta}$ to be symmetric to the lift-off angle about vertical axis,

$$\bar{\theta} \quad = \quad -\theta_{lo}. \tag{11}$$

Then, from (6) we obtain desired touch-down angular velocity $\bar{\dot{\theta}}$ as,

$$\begin{aligned}\dot{\theta}_{td} &= -\frac{1}{r_0}(\dot{x}_{td-}\cos\theta_{td} + \dot{z}_{td-}\sin\theta_{td}) \\ &= -\frac{1}{r_0}(\dot{x}_{lo}\cos\theta_{lo} + \dot{z}_{lo}\sin\theta_{lo}) \\ &= \dot{\theta}_{lo} =: \bar{\dot{\theta}}. \end{aligned}\tag{12}$$

Additionally, considering

$$\dot{z}_{td+} \quad = \quad \dot{z}_{td-} = \dot{z}_{lo} - gT_v \tag{13}$$
$$\dot{x}_{td+} \quad = \quad \dot{x}_{td-}, \tag{14}$$

we obtain

$$\begin{aligned}\dot{r}_{td+} &= (\dot{z}_{lo} - gT_v)\cos\theta_{td} - \dot{x}_{lo}\sin\theta_{td} \\ &= -\dot{z}_{lo}\cos\theta_{lo} + \dot{x}_{lo}\sin\theta_{lo} \\ &= \dot{r}_{lo}. \end{aligned}\tag{15}$$

Therefore, the condition (7) is ensured if and only if $\dot{r}_{lo} > 0$.

Again, note that (10) and (11) is no more than the one of the options.

4.2 Derivation of the controller

Using new variables:

$$\psi \quad = \quad \theta - \phi, \tag{16}$$
$$\sigma \quad = \quad J_b\phi + J_l\theta, \tag{17}$$

the dynamics of flight phase can be rewritten as,

$$\begin{cases} \ddot{x} = 0 \\ \ddot{z} = -g \\ \ddot{\psi} + \Omega_h^2\psi = -\frac{\Omega_h^2}{K_h}\tau \\ \ddot{\sigma} = 0. \end{cases} \tag{18}$$

Since these are the independent second order linear ODEs, we can easily dead-beat ψ and $\dot{\psi}$, by once-switching of the constant inputs.

Discretizing (18) using the piecewise constant inputs,

$$\tau \quad = \quad \begin{cases} \tau_1, & \text{if } \; 0 \le t < T_v/2 \\ \tau_2, & \text{if } \; T_v/2 \le t < T_v, \end{cases} \tag{19}$$

where t is the time after the lift-off, and integrating (18), the control inputs can be calculated as follows.

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = B(T_v)^{-1}\left(\begin{bmatrix} \bar{\psi} \\ \bar{\dot{\psi}} \end{bmatrix} - A(T_v)\begin{bmatrix} \psi_{lo} \\ \dot{\psi}_{lo} \end{bmatrix}\right), \tag{20}$$

where,

$$\begin{bmatrix} \bar{\psi} \\ \bar{\dot{\psi}} \end{bmatrix} = \frac{1}{J_b}\left((J_b + J_l)\begin{bmatrix} \bar{\theta} \\ \bar{\dot{\theta}} \end{bmatrix} + C(T_v)\begin{bmatrix} \sigma_{lo} \\ \dot{\sigma}_{lo} \end{bmatrix}\right), \tag{21}$$

and,

$$\begin{aligned} A(T_v) &= \begin{bmatrix} \cos(\Omega_h T_v) & \frac{1}{\Omega_h}\sin(\Omega_h T_v) \\ -\Omega_h\sin(\Omega_h T_v) & \cos(\Omega_h T_v) \end{bmatrix} \\ B(T_v) &= \frac{1}{K_h}\begin{bmatrix} \cos(\Omega_h T_v) & \cos(\frac{\Omega_h T_v}{2}) \\ -\Omega_h\sin(\Omega_h T_v) & -\Omega_h\sin(\frac{\Omega_h T_v}{2}) \end{bmatrix} \\ C(T_v) &= \begin{bmatrix} 1 & T_v \\ 0 & 1 \end{bmatrix}. \end{aligned}$$

Since $\det(B(T_v)) = \frac{\Omega_h}{K_h^2}\sin(\frac{\Omega_h T_v}{2})$ holds, control inputs (20) always exist, unless $T_v = 0$, or $T_v = \frac{2\pi}{\Omega_h}$.

4.3 Simulation results and discussion

We have simlulated the action of the *Non-Dissipative Touch-Down Control* (19), (20) and (21), together with (10)-(12), for wide range of initial conditions. **Figure 3** depicts the time evolutions of state variables $(\dot{x}, \theta, \phi, z)$ and the energy level E, and the stepwise inputs (τ_1, τ_2). The lower three small graphs represent the selected images of the Poincaré map. Here, the free parameters of initial values are chosen to be $\theta_0 = 0.25$[rad] and $\dot{x}_0 = 2$[m/s]. This corresponds to middle speed of hopping for the robot of **Table 3**.

We can see from the images of the Ponincaré map that the obtained gaits are quasi-periodic ones that can be seen in some Hamiltonian systems [14]. Their periods depend on the initial conditions.

5 Stabilization of periodic orbits

5.1 Stabilization to a desired periodic orbit by adaptation of touch-down angle

As mentioned in the previous section, if the solution eventually converges to the 1-periocic orbits, then the inputs also converge to the constant values. What should we do to change the period ? Remember Section 4.1, where we said the choice of the desired touch-down angle is not unique.

We invented the following stepwise adaptation law:

$$\overline{\theta(k)} = \begin{cases} -\frac{1}{2}(\theta_{lo}(k) + \theta_{lo}(k-p)), & \text{if } k > p \\ -\theta_{lo}(k), & \text{else.} \end{cases} \tag{22}$$

Here, $k \ge 1$ is the iteration step, and $p \ge 1$ is the desired period.

We also note that this method is similar to Pyragas's method [15] to stabilize some chaotic orbits to the desired periodic orbit, if we consider the touch-down angle as a control inputs to the discrete dynamical system (Poincaré map of the system).

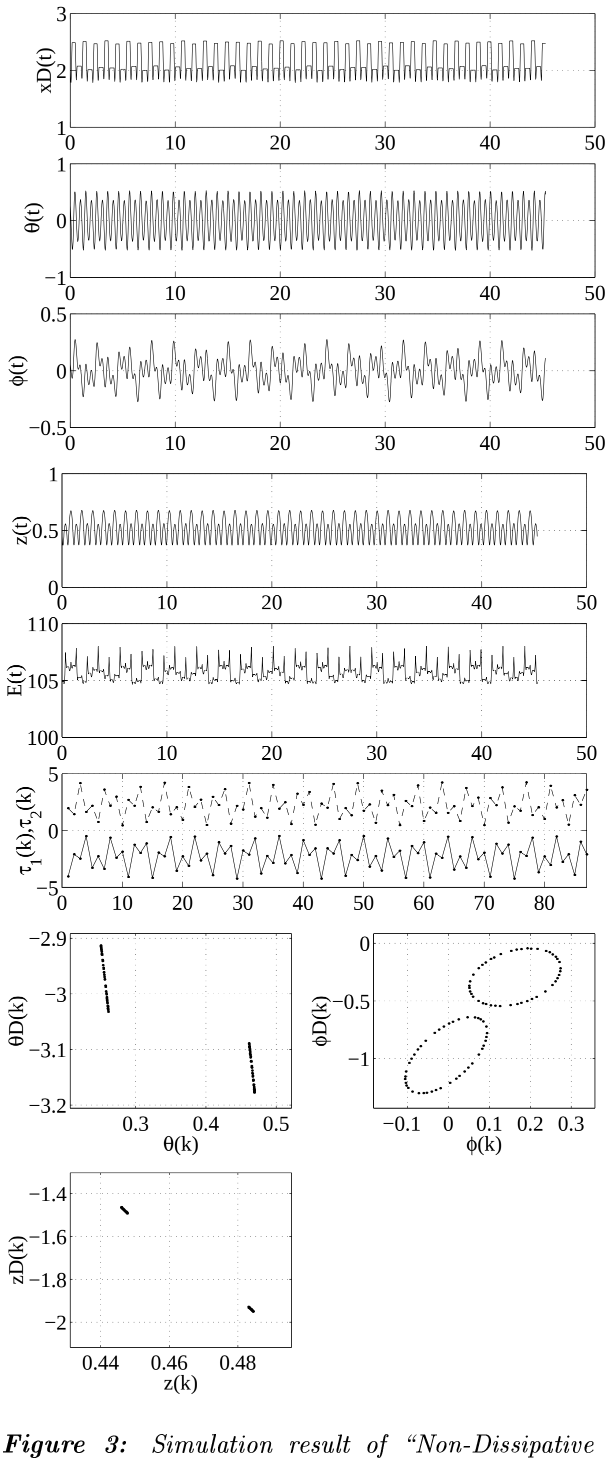

Figure 3: *Simulation result of "Non-Dissipative Touch-Down Control". The lower four small graphs represent the selected images of the Poincaré map. From this, you can see the gait obtained is quasi-periodic one.*

5.2 Input minimization by adaptation of spring stiffness

During walking or running, animal may adapt their joint stiffness by varying the tension of the antagonistic muscles to minimize energy consumption. In order to simulate such a adaptation mechanism, we examine an adaptation law for the stiffness of the hip spring to minimize the control inputs.

Since we are using piecewise constant inputs, the simplest adaptation law will be given as:

$$\Omega_h(k+1) = \begin{cases} \Omega_h(k) + \gamma\left\{\tau_2(k) - \tau_1(k)\right\}, & \text{if } k > 2 \\ \sqrt{K_h(\frac{1}{J_b} + \frac{1}{J_l})}, & \text{if } k = 1, \end{cases} \tag{23}$$

where, $\gamma > 0$ is the adaptation gain.

5.3 Simulation results and discussion

Because of the rack of the space, only the result of the stabilization to 1-periodic gait is shown. **Figure 4** shows the time evolution, where the touch-down angle adaptation is activated from the beginning, while the spring stiffness adaptation is activated from the 60th step for visibility. The initial conditions are the same as those of **Figure 3**. We have drawn the line between the points (images) of the Poincaré map, for visibility. We can see from the figure that the adaptation laws works well and control inputs eventually converges to zero ! Thus, we have succeeded in complete passive running. It is not surprising that phase volume is contracting because the system is not actually Hamiltonian.

We have also simulated stabilization to an arbitrary periodic gaits and confirmed that the periods of solutions converge to the desired one. But the complete passive running has not yet been succeeded for multi-periodic gait. We also observed there are significant difference in convergence speed, depending on the initial conditions.

6 Conclusion

We presented a novel controller for stabilization of a passive one-legged hopping robot. First, based on the dynamics of this nonlinear hybrid system, we derived a simple control law, *Non-Dissipative Touch-Down Control*, to ensure the energy preservation and continuation. The generated hopping gaits were found to be quasi-periodic orbits, which can be seen in some Hamiltonian systems. Next, we proposed a simple parameter adaptation law to asymptotically stabilize the quasi-periodic gaits to the periodic gaits of arbitrary period, and spring stiffness adaptation law to minimize control inputs. Simulation results demonstrated that the robot can eventually hop without any control inputs, especially for 1-periodic gait. Our current study includes robustness analysis, derivation of tracking controllers, and the application to the one-legged robot "Kenken" [5], or to the biped and quadruped robot under construction.

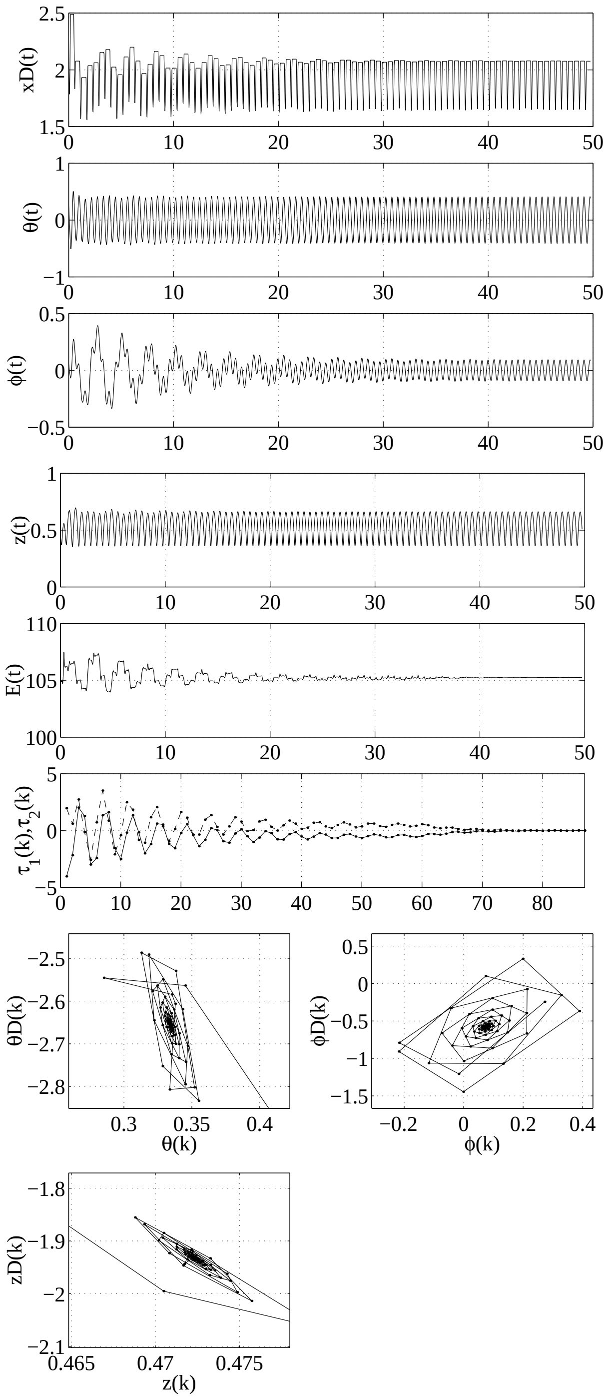

Figure 4: *Simulation results of the stabilization to 1-periodic gait and the adaptation of spring stiffness. Control inputs eventually converge to zero.*

Acknowledgments

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